# Chronotension Field Theory — Soliton Derivation Formulas

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Purpose: Formal derivation of soliton behavior in Chronotension Field Theory (CFT) via the sine-Gordon equation.

## 1. CFT Lagrangian (Scalar Field Form)

\[  
\mathcal{L}\_{\text{CFT}} = -\frac{1}{2} \mathcal{T}(x, t) \, \partial^\mu \eta \, \partial\_\mu \eta - V(\eta)  
\]  
Where:  
- \(\mathcal{T}(x,t)\): local tension scalar field  
- \(\eta(x,t)\): viscosity field (acts as scalar field)  
- \(V(\eta)\): potential energy of the time field

## 2. Euler–Lagrange Equation

\[  
\partial\_\mu \left( \frac{\partial \mathcal{L}}{\partial (\partial\_\mu \eta)} \right) - \frac{\partial \mathcal{L}}{\partial \eta} = 0  
\]  
Assuming constant tension \(\mathcal{T}(x,t) = T\_0\), we simplify:  
\[  
T\_0 (\partial\_t^2 \eta - \partial\_x^2 \eta) + \frac{dV}{d\eta} = 0  
\]

## 3. Sine-Gordon Potential

To match the sine-Gordon equation:  
\[  
\partial\_t^2 \eta - \partial\_x^2 \eta + \sin(\eta) = 0  
\]  
We require:  
\[  
\frac{dV}{d\eta} = -T\_0 \cdot \sin(\eta) \Rightarrow V(\eta) = T\_0 \cdot \cos(\eta)  
\]  
Thus, the Lagrangian becomes:  
\[  
\mathcal{L}\_{\text{SG-CFT}} = -\frac{1}{2} T\_0 \, (\partial^\mu \eta \, \partial\_\mu \eta) - T\_0 \cdot \cos(\eta)  
\]

## 4. Soliton Solution (Kink)

A single-kink soliton solution is:  
\[  
\eta(x,t) = 4 \arctan \left( \exp\left( \pm \gamma(x - vt - x\_0) \right) \right)  
\]  
Where:  
- \(v\): velocity of the soliton  
- \(\gamma = 1/\sqrt{1 - v^2}\): Lorentz factor  
- \(x\_0\): center position of the kink

## 5. Energy Density

\[  
\mathcal{E}(x) = \frac{1}{2} T\_0 \left( (\partial\_t \eta)^2 + (\partial\_x \eta)^2 \right) + T\_0 \cdot (1 - \cos(\eta))  
\]

## 6. Topological Charge (Q)

For soliton stability:  
\[  
Q = \frac{1}{2\pi} \int\_{-\infty}^{\infty} \partial\_x \eta(x) \, dx = \pm 1  
\]

Conclusion: This confirms that chronodes can be fully described by soliton solutions when CFT is endowed with a sine-Gordon-type potential. Chronodes are therefore topologically stable structures in the viscosity field η(x,t), supported by tension-mediated dynamics.